Clinical decision-making, using Bayes Theorem and statistics derived from contingency tables, including sensitivity, specificity, and negative and positive predictive values

OSCTR
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Statistical concepts for clinical investigators

2x2 table that relates test results to an outcome or disease state

		True state, sometimes validated by "gold standard"		
		Outcome present (+)	Outcome absent (-)	
Test Result	positive test (+)	TP	FP	total who test positive
	negative test (-)	FN	TN	total who test negative
		total with condition	total without condition	total

Prevalence = (TP+FN)/ total, the proportion who truly have the condition.

Important conditional probabilities:

Sensitivity and specificity.

Sensitivity = TP/(TP+FN) = $p(T^+ \mid O^+) = p(O^+ \cap T^+) / p(O^+)$

the probability that someone with the condition will test positive.

Specificity = TN/(TN+FP) = $p(T^- \mid O^-) = p(O^- \cap T^-) / p(O^-)$

The probability that someone without the condition will test negative.

Because sensitivity is calculated using data only from those in whom the outcome is present, and specificity using only those in whom it is absent, these measures are considered to be *stable properties of a test*.

Predictive Values.

Predictive Value Positive (PV+) = TP/(TP+FP) = $p(O^+ \mid T^+) = p(O^+ \cap T^+) / p(T^+)$

The probability that someone with a positive test truly has the condition.

Predictive Value Negative (PV-) = TN/(TN+FN) = $p(O^- \mid T^-) = p(O^- \cap T^-) / p(T^-)$

the probability that someone with a negative test truly does not have the condition.

Because predictive values are calculated using information from BOTH those in whom the outcome is present, and from those in whom it is absent, these measures are *sample-specific, and depend on the prevalence of the outcome* in the population to which the test is applied.

Bayes theorem unifies these conditional probabilities

Bayes theorem describes relationships among certain conditional probabilities, including the fact that that sensitivity and PV+ share the same numerator. Using that fact, and some judicious substitutions, we can express predictive values in terms of sensitivity, specificity *and prevalence*.

Predictive Value Positive (PV+) $= p(O^+ \cap T^+) / p(T^+)$

$= [Sn * p(O^+)] / \{p(T^+ \cap O^+) + p(T^+ \cap O^-) \}$

$= [Sn * p(O^+)] / \{ Sn * p(O^+) + [1-p(T^- | O^-)]*p(O^-)\}$

Replacing $p(O^+)$, the probability of the outcome, with prevalence,

and $p(O^-)$ with 1-prevalence,

$$PV+ = \frac{Sn * prev}{Sn*prev + (1-Sp)*(1-prev)}$$

In certain settings, we regard the PV+ as a post-test probability (PostTP), the probability of an outcome given the result of a test.

In these settings, we can substitute, for $p(O^+)$ or prevalence, a "pre-test probability" (preTP), a judgment about the probability of the outcome BEFORE administering the test. Then:

$$PostTP = \frac{Sn * prePT}{Sn*prePT + (1-Sp)*(1-prePT)}$$

Similar algebra produces an expression for the predictive value negative (PV-), the probability that a patient whose test is negative does NOT have the disease or condition in question.

$$PV- = \frac{Sp * (1-prev)}{Sp \times (1-prev) + (1-Sn) \times prev}$$

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March 8, 2014